PRICING OF ATC/ATM SERVICES
PART II: THE CASE OF A PRIVATE PROVIDER

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RESUMO
Neste estudo uma abordagem multi niveis é proposta para estudar o problema da definição das taxas a serem aplicadas por un fornecedor privado de serviços de controle e gerenciamento do trafego aéreo. Esta analise leva a considerar a resolução de um problema de programação matemática de tipo bi nivel onde o provedor do serviço é o "líder" e o setor das empresas aéreas é o "seguidor". No caso mono dimensional, uma solução analítica é estabelecida e analizada para este problema. Esta solução é comparada à solução obtida quando o provedor de serviço em vez de procurar a maximização do seu lucro procura fomentar a demanda final dos usuários do sistema de transporte aéreo. Um esquema natural de negociação entre o provedor de serviço e as empreasas aéreas é então considerado e condições suficientes de convergência do processo para a solução ôtima são estabelecidas.

ABSTRACT
In this study a multilevel framework is proposed to analyze the charges definition problem for a private ATC/ATM service provider. This leads to the formulation of a bi-level optimization program involving ATC/ATM service provider as the leader and the whole airline sector as the follower. A direct optimal solution has been obtained in the simplistic considered one dimensional case. This solution is compared with the one obtained in the case of a public service provider maximising the level of end users demand. Then a natural negotiation process between the ATC/ATM service provider and the airline sector is introduced. Sufficient conditions are established so that the proposed negotiation process converges towards the optimal solution.
Keywords: pricing, bilevel programming, ATC/ATM, air transportation

1 INTRODUCTION

This study follows the study about pricing in the case of a public ATC/ATM service provider. It proposes a similar multilevel framework to analyze the ATC/ATM charges definition problem in the case of a private ATC/ATM service provider in a partially deregulated market. The main assumptions with respect to costs and constraints are the same than in the paper devoted to the public ATC/ATM service provider case. Here also, for sake of simplicity, only the one dimensional case is considered. The objective assumed for the ATC/ATM service provider and the airlines sector is profit maximization. Here again the airline sector is taken as a whole, so that market competition between airlines is not contemplated. An air travel demand function, reactive to airlines tariffs is introduced to take into account indirect influence of ATC/ATM charges on passengers demand levels. Constraints are considered so that the economic performance of the airline sector is not impaired by the retained levels of ATC/ATM charges. This analysis leads to the formulation of a bi-level optimization program involving ATC/ATM service provider as the leader and the whole airline sector as the follower.

The solution of this problem is discussed following a direct analytical approach, then conditions such that its solution produces a same level of demand by end users than in the case of a public ATC/ATM service provider are discussed. Then a new negotiation process between the ATC/ATM service provider and the airlines sector is introduced and its convergence is studied while its solution is compared to the optimal one.

2 BASIC ASSUMPTIONS AND RESULTS

Here again we consider the case of an elementary air transportation system composed of a single pair of airports linked by a single air route, with a unique ATC service provider and a unique airline operating between these two airports.

Recall that the potential demand is supposed to be composed of round trips and to obey, for the sake of simplicity and the ability to develop clear analytical and graphical results, to the following affine demand function:

$$\phi = D_0 - \rho\, \pi(1+\lambda) \tag{1}$$

where ϕ is the effective level of potential demand. Here it is supposed that there is a unique class of travelers and a unique apparent price $\tilde{\pi} = \pi(1+\lambda)$ is adopted for round trips. D_0 is an absolute potential demand and ρ is a constant positive parameter characteristic of the response of the market to price changes. The parameter λ represents a tax index applied to each trip ticket.

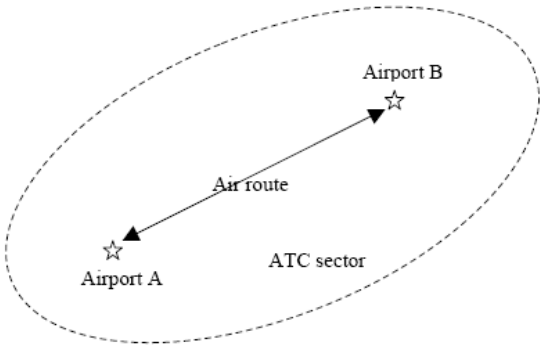


Figure 1. The elementary ATC/ATM case

Beyond the ATC/ATM charges, an additional way to fund the ATC/ATM services which appears natural, is to assign a proportion α ($\alpha \in [0,1]$) of the above tax, which is paid by the final users, to the ATC service provider.

The transport capacity of the airline is given by the maximum affordable frequency of service $f_{\max}$ which is related with the size of the fleet of the airline. Here for simplicity and considering that for the given time period *f* can be a high number, *f* will be taken as real.
The airlines operations costs are again given by:

$$(c+v)\, f + C^F_{ALN} \tag{2}$$

where c is a positive parameter related with the price of fuel, the cost of the crew and the length of the flights; C^F_{ALN} is a fixed cost related with the sizes of the fleet and the crews of the airline as well as with the characteristics of the operated network; *v* is the ATC/ATM tariff

applied to a round flight, including en route control and airport control. Again, no distinction will be made between airport taxes and approach and en route charges.
The available seat capacity is given by:

$$q\ f \tag{3}$$

where q is the mean seat capacity of an aircraft of the fleet of the airline.

Here it is supposed that the objective of the private ATC/ATM service supplier is to maximize its profit while guarranteeing a minimum economic return R_{ALN} for the airline so that she maintains her activity. The public authority collects a tax (rate λ) over each trip and may refund for an amount μ the ATC/ATM service supplier. The operating costs of the private ATC service supplier are given now by :

$$s\ f + Q_{ATC}^{F} \tag{4}$$

here:

- s is a positive parameter (a mean variable cost with respect to frequency). It is related mainly with the length of the flights.
- Q_{ATC}^{F} is a fixed cost related with the sizes of the airspace and of the ATC/ATM staff.

The private ATC/ATM service supplier is reputed to be more cost effective than the public one, such that:

$$s \le \sigma \qquad \text{and} \qquad Q_{ATC}^{F} \le C_{ATC}^{F} \tag{5}$$

The operating costs of the private ATC supplier are given by:

$$\sigma\ f + C_{ATC}^{F} \tag{6}$$

here σ is a positive parameter (a mean variable cost with respect to frequency). It is related mainly with the length of the flights; C_{ATC}^{F} is a fixed cost related with the characteristics of the controlled airspace and with the size of the ATC/ATM staff. Again, no saturation effects with consequences over the cost functions of the ATC/ATM service provider and the airline are considered in this study.

As shown in the previous paper, the solution of the airline's profit maximization problem:

$$\max_{\pi,\ f} \quad \pi\ \phi - ((c+v) f + C_{ALN}^{F}) \tag{7}$$

with

$$\phi = \max\{0, \min\{q\ f,\ D_0 - \rho\ \pi(1+\lambda)\}\ \} \tag{8}$$

and

$$0 \le f \le f_{\max} \tag{9}$$

is given by:

$$f^{*} = \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v) \tag{10}$$

$$p_{ALN}^{\max} = \frac{1}{\tilde{\rho}}\left(\frac{(c+v)\tilde{\rho} + qD_0}{2q}\right)^2 - (c+v)D_0 / q - C_{ALN}^{F} \tag{11}$$

$$\pi^{*} = \frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}} \tag{12}$$

with

$$\phi = \frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v) \tag{13}$$

3 OPTIMAL PRICING FOR A PRIVATE ATC/ATM SERVICE SUPPLIER

It is supposed that the objective of the private ATC/ATM service supplier is to maximize his profit while guarranteeing a minimum economic return R_{ALN} for the airlines sector so that he maintains his activity. It is assumed that at the same time the airline tries to maximize her benefit taking into account her cost function and the ATC/ATM tariff. The public authority collects a tax (rate λ) over each trip and may refund with an amount μ the ATC/ATM service supplier. The operating costs of the private ATC/ATM service supplier are given now by $s\ f + Q_{ATC}^F$ where *s* is a positive parameter (a mean variable cost with respect to frequency). It is related mainly with the length of the flights. Q_{ATC}^F is a fixed cost related with the sizes of the airspace and of the ATC/ATM staff. According to the above assumptions, a bilivel program can be established:

$$\max_{v \geq 0} vf - (s\ f + Q_{ATC}^F) \tag{14}$$

where the profit of the ATC/ATM service provider is given by:

$$r_{ATC} = vf - (s\ f + Q_{ATC}^F) + \mu \tag{15}$$

with an exogenous refunding constraint:

$$\mu \leq \lambda\ \pi\ \phi \tag{16}$$

and the airline's profit constraint:

$$\pi\ \phi - ((c+v)f + C_{ALN}^F) \geq R_{ALN} \tag{17}$$

where π and *f* are given by the solution of the optimization problem defined by relations:

$$\max_{\pi,\ f} \quad \pi\ \phi - ((c+v)f + C_{ALN}^F) \tag{18}$$

$$\text{with} \qquad \phi = \max\{0, \min\{q\ f, D_0 - \rho\ \pi(1+\lambda)\}\} \quad \text{and} \quad 0 \leq f \leq f_{\max} \tag{19}$$

Taking into account the behaviour of the airline, the optimization problem of the private ATC/ATM service supplier becomes:

$$\max_{v \geq 0} \quad (v-s)(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v)) - Q_{ATC}^F \tag{20}$$

$$\text{under} \quad (\frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}})(\frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v)) - ((\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v))(c+v) + C_{ALN}^F) \geq R_{ALN} \tag{21}$$

$$\text{and} \qquad 0 \leq v \leq (q/\rho)\tilde{D}_0 - c \tag{22}$$

The unconstrained solution of the above problem is given by:

$$\hat{v} = \frac{s}{2} + \frac{q}{2\tilde{\rho}} D_0 \tag{23}$$

Let us consider the roots of equation:

$$av^2 - bv + c = 0 \tag{24}$$

$$\text{with} \begin{cases} a = \tilde{\rho}/(4q^2), \\ b = (D_0 - c\,\tilde{\rho}/q)/(2q) \\ c = \dfrac{D_0^{\,2}}{4\tilde{\rho}} + \dfrac{\tilde{\rho}\,c^2}{4q^2} - \dfrac{cD_0}{2q} - C_{ALN}^F - R_{ALN} \end{cases} \tag{25}$$

Here we assume that R_{ALN} and λ are chosen such as $p_{ALN}(0) \geq R_{ALN}$. If the following conditions $4\,a\,c - b^{\,2} > 0$ and $0 \leq \hat{v} \leq (q/\tilde{\rho})D_0 - c$ are satisfied, then (see figure 2):

$$v^* = \hat{v} \tag{26}$$

where $\hat{v}$ which is given by (20) is solution of the optimization problem of the private ATC service supplier. If condition $p_{ALN}(0) \geq R_{ALN}$ is satisfied while $0 \leq \hat{v} \leq (q/\tilde{\rho})D_0 - c$ is not satisfied, the solution of the optimization problem of the private ATC service supplier is given by:

$$v^* = v_{\max} = (q/\tilde{\rho})D_0 - c \tag{27}$$

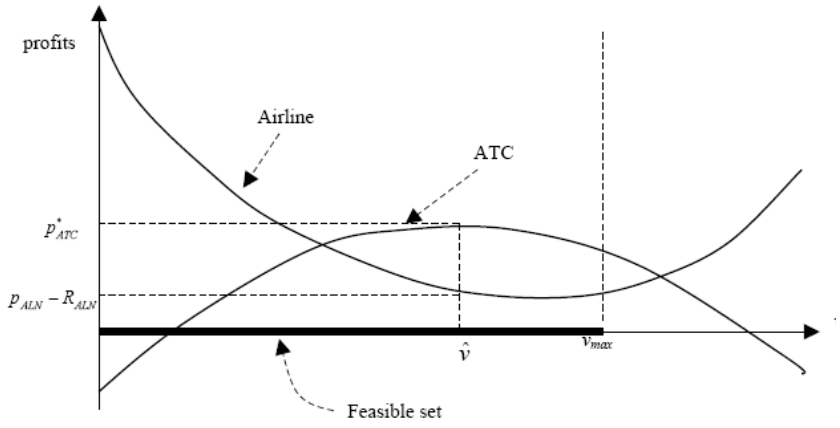


Figure 2. Solution of the private ATC/ATM supplier problem (no real roots for (24))

When condition $p_{ALN}(0) \geq R_{ALN}$ is satisfied while condition $4\,a\,c - b^{\,2} > 0$ is not satisfied, equation (24) s two positive roots v^- and v^+ with $v^- \leq v^+$. Here we can consider the three different cases (see figure 3):

if $v^- \leq \hat{v}$ and $v^+ \geq \hat{v}$ then

$$\textit{if} \quad v^+ > v_{\max} \ \textit{then} \quad v^* = v^- \tag{28-a}$$

$$\textit{if} \quad v^+ \leq v_{\max} \ \textit{then} \quad v^* = \max_{v \in \{v^-, v^+\}} p_{ATC}(v) \tag{28-b}$$

$$\text{if } v^- \le \hat{v} \text{ and } v^+ \le \hat{v} \text{ then } v^* = \hat{v} \tag{29}$$

$$\text{if } v^- \ge \hat{v} \text{ then } v^* = v^- \tag{30}$$

where: $v^- = (b - \sqrt{b^2 - 4ac})/(2a)$ and $v^+ = (b + \sqrt{b^2 - 4ac})/(2a)$ (31)

The level of the satisfied demand is given by:

$$\phi^* = \frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c + v^*) \tag{32}$$

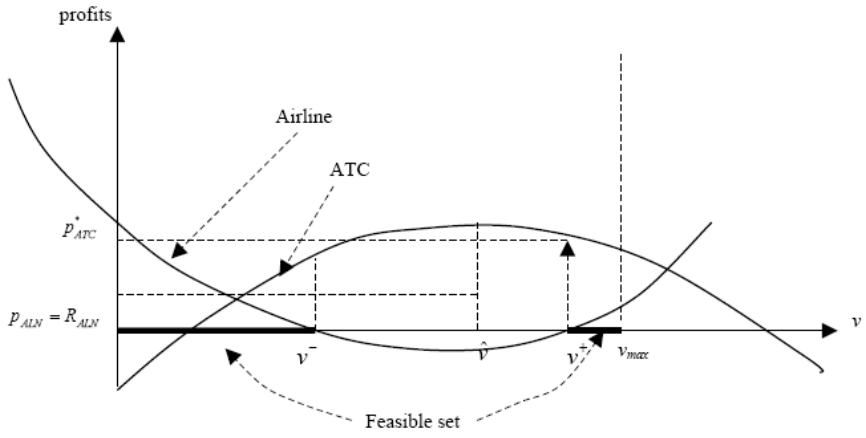


Figure 3. Solution of the private ATC/ATM supplier problem (two real roots for (24))

The profit of the private ATC service supplier is equal to:

$$p^*_{ATC}(\mu) = -\frac{\tilde{\rho}}{2q^2}v^{*2} + \left(\frac{D_0}{2q} + \frac{s\tilde{\rho}}{2q^2}\right)v^* + \left(\frac{\tilde{\rho}\, s\, c}{2q^2} - \frac{s\, D_0}{2q} - Q^F_{ATC} + \mu\right) \tag{33}$$

Refunding μ is chosen in order to guarrantee that $p^*_{ATC} \ge R_{ATC}$.

$$\textit{if} \quad \left(-\frac{\tilde{\rho}}{2q^2}v^{*2} + \left(\frac{D_0}{2q} + \frac{s\tilde{\rho}}{2q^2}\right)v^* + \left(\frac{\tilde{\rho}\, s\, c}{2q^2} - \frac{s\, D_0}{2q} - Q^F_{ATC}\right)\right) \ge R_{ATC} \quad \textit{then} \quad \mu = 0 \tag{34}$$

$$\begin{aligned} &\textit{if} \quad \left(-\frac{\tilde{\rho}}{2q^2}v^{*2} + \left(\frac{D_0}{2q} + \frac{s\tilde{\rho}}{2q^2}\right)v^* + \left(\frac{\tilde{\rho}\, s\, c}{2q^2} - \frac{s\, D_0}{2q} - Q^F_{ATC}\right)\right) < R_{ATC} \\ &\textit{then} \qquad \mu = \min\left\{R_{ATC} - p^*_{ATC}(0), \lambda\phi^*\right\} \end{aligned} \tag{35}$$

where $\phi^* = \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^*)$. Here, the profit of the airline is given by:

$$p^* = \frac{1}{\rho(1+\lambda)}\left(\frac{(c+v^*)\rho + qD_0}{2q}\right)^2 - (c+v^*)D_0/q - C^F_{ALN} \tag{36}$$

which is superior or equal (see figures 2 and 3) to the minimum guarranted level R_{ALN}.

4 REGULATION SCHEME WITH A PRIVATE ATC/ATM SERVICE PROVIDER

From the solution of the airline's profit maximization problem it appears that to get the same level of satisfied demand in the two cases (public or private ATC/ATM service providers), it is necessary to apply the same ATC/ATM tariff to the airline operations. The ATC/ATM tariff in the public supplier case is given by :

$$v_{pub} = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(1)}}{2+\lambda} - c \tag{37}$$

while in the private case, it is given by:

$$v_{pri} = v^* \tag{38}$$

where , according to the case, as seen in section IV, v^* adopts different expressions. In general the resulting tariffs will be such as:

$$v_{pri} \geq v_{pub} \tag{39}$$

The proposed refunding scheme in section IV has in fact no effect on the level of the tariff adopted by the private ATC/ATM service provider and then on the resulting level of the satisfied demand. Then another scheme must be conceived. What is proposed here is to provide a subvention to the airline by lowering the effective tariff payed to the private ATC/ATM service provider, the difference being compensated by the state up to the amoint collected directly from the passengers through the passenger tax.

Let w be the part of the ATC/ATM tariff which is subventionned by the state. This value should be such as:

$$w = v_{pri} - v_{pub} \tag{40}$$

with the condition:

$$w\, f^* + \mu \leq \lambda\, \phi^* \tag{41}$$

or

$$w\left(\frac{D_0}{2q} - \frac{\rho(1+\lambda)}{2q^2}(c + v_{pub})\right) + \mu \leq \lambda\left(\frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c+v_{pub})\right) \tag{42}$$

Then, it will be possible to get the same level for the satisfied demand if:

$$\lambda \geq (v_{pri} - v_{pub})/q \tag{43}$$

Then we get the structure for financial flows displayed on figure 4.

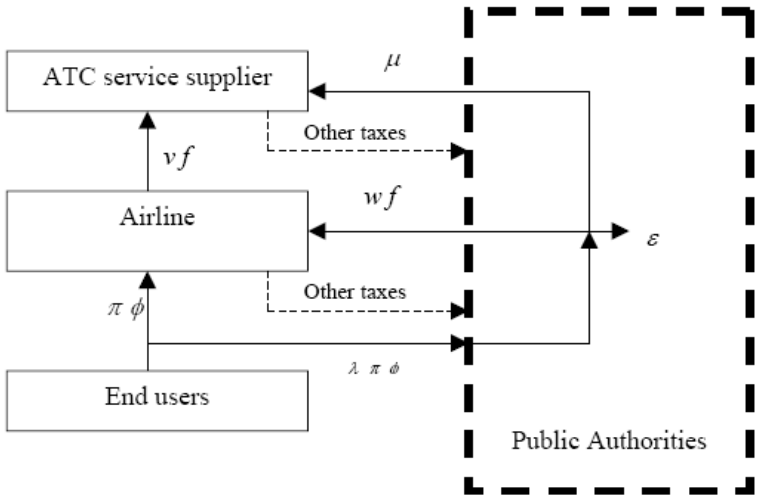


Figure 4. Financial flows and activity levels with private supplier with regulation

5 NEGOTIATION PROCESS WITH AIRLINES

Following the general formulation, a natural negociation process based on the objectives of the involved economic agents could be the following:

The private ATC/ATM service supplier solves at iteration n+1 the following problem with respect to v , given π^n and f^n:f

$$v^{n+1} = \arg\{ \max_{v} \quad v f^n + \alpha \lambda \pi^n \phi^n - (s f^n + Q_{ATC}^F) \} \tag{44}$$

with the airline's profit constraint:

$$\pi^n \phi(v) - ((c+v) f^n + C_{ALN}^F) \geq R_{ALN} \tag{45}$$

where π^n and f^n are provided by the airline sector which solves problem (18), (19) given v^n:

This process is represented in figure 5.
Supposing that v will be chosen superior to s, the private ATC supplier problem reduces here to the finding of the maximum value of v which satisfies the airline budget constraint:

$$v^{n+1} = \frac{\pi^n \phi^n - (R_{ALN} + C_{ALN}^F + c f^n)}{f^n} \tag{46}$$

Here also we consider the case in which the solution of the airline problem is given by (10) , (11) and (12). Then we get a recurent formula for v^n:

$$v^{n+1} = \frac{1}{2}(D_0 \frac{q}{\tilde{\rho}} + c) + \frac{1}{2} v^n - \frac{(2q^2/\tilde{\rho})(R_{ALN} + C_{ALN}^F)}{(D_0 \frac{q}{\tilde{\rho}} - c) - v^n} \tag{47}$$

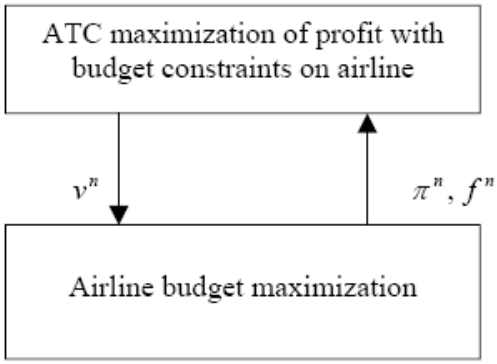


Figure 5. Negociation process between private ATC/ATM supplier and airlines

Here also we consider that condition (35) is satisfied, then in figure 6, the convergence of the negociation process is analyzed graphically.

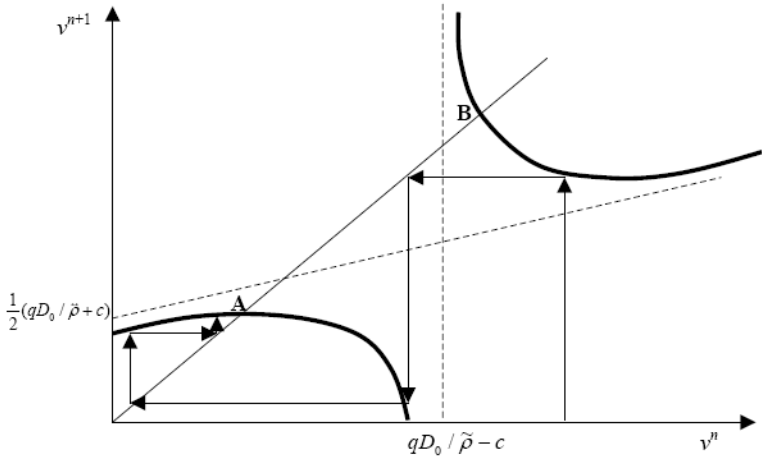


Figure 6. Convergence of the private ATC/ATM- Airline negociatio.. process

It appears that if:

$$c > 2(q/\tilde{\rho})\sqrt{C^F_{ALN} + R_{ALN}} \tag{48}$$

or

$$R_{ALN} < c^2\,\tilde{\rho}^2/4q^2 - C^F_{ALN} \tag{49}$$

then there are two equilibrium points: point A which is a stable equilibrium solution reached from lower v values and point B which is an unstable equilibrium solution where:

$$v_A = qD_0/\tilde{\rho} - \sqrt{c^2 - 4\frac{q^2}{\tilde{\rho}^2}(C^F_{ALN} + R_{ALN})} \quad \text{and} \quad v_B = qD_0/\tilde{\rho} + \sqrt{c^2 - 4\frac{q^2}{\tilde{\rho}^2}(C^F_{ALN} + R_{ALN})} \tag{50}$$

It appears that point A corresponds to the optimal solution of the bilevel problem found previously.. Moreover, it appears that condition (49) is satisfied by the problem parameters if

a negative return is accepted by the private ATC service provider. When condition (49) is not satisfied, the negociation process does not converge to an equilibrium point.

6 CONCLUSION

In this study a multilevel framework has been proposed to analyze the ATC/ATM charges definition problem in the case of a private ATC/ATM service provider in a deregulated market. Here the one dimensional case has been considered. The objective which has been assumed for the ATC/ATM service provider as well as the airlines sector is profit maximization. This has led to the formulation of a bi-level optimization program involving ATC/ATM service provider as the leader and the whole airline sector as the follower. A direct optimal solution has been obtained in the simplistic considered one dimensional case. Conditions such that its solution produces a same level of demand by end users than in the case of a public ATC/ATM service provider are established.Then a natural negotiation process between the ATC/ATM service provider and the airline sector has been introduced, splitting this problem in two dependent problems: one where ATM authorities determine ATC/ATM charges for given airlines tariffs and one where the airline sector determines the tariffs and seat capacity supplies over the different markets for a given ATC/ATM charge level. Sufficient conditions have been established so that the proposed negotiation process converges towards the optimal solution. Then here again, it appears of interest to the service provider to adopt a similar negotiation process when dealing with the full scale networked pricing problem.

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