NETWORK SLOT ALLOCATION FOR DOMESTIC TRAFFIC MANAGEMENT

Felix A. C . Mora-Camino
MAIAA, Ecole Nationale de l'Aviation Civile, Toulouse, France
Tel. :+3362174358 E-mail: felix.mora@enac.fr

Li Weigang
TransLab, Department of Computer Science, University of Brasilia, Brasilia, Brazil
Tel.: +55 61 3107 3679 E-mail address: weigang@unb.br

Oscar Díaz Olariaga
Facultad de Ingeniería Civil, Universidad Santo Tomás, Bogotá, Colombia
Tel.: +57-304-582-0103 E-mail: OscarDiazOlariaga@usantotomas.edu.co

ABSTRACT

In this paper is considered the problem of managing traffic capacity at network level through slot allocation. This problem can be seen either as an extension of slot allocation from an airport to a network of airports or a specialization of Ground Delay Programs as developed in USA. After introducing slot allocation theory, a new approach to assess cumulative delays for airlines is developed and an equivalent delay multiplier is proposed. This new index allows to rank more effectively candidates flights during a slot allocation process. This is illustrated in the case of a compression algorithm for slot allocation.

Keywords: air traffic management, delays in networks, airport capacity, slot allocation.

1 INTRODUCTION

Congestion in transportation systems is considered to be a limiting factor to economic growth. Many transportation systems are composed of a discrete set of companies working between a limited set of transportation terminals where endogenous congestion affects individual arrival and departure operations. This is particularly the case when considering a domestic network of airports linked together by an air transportation system. Collaborative Decision Making (CDM) has been developed recently to deal with the coordination of activities addressed to the processing of vehicles between their arrival and departure at an airport. This concept favors terminal overall efficiency by implementing on-time communication of information and predictions between the different terminal operators which process the vehicle and the payload (passengers or freight) at the terminal. However this approach has not taken into consideration network effects generated by travel perturbations or by disruptions at some airport.

In this paper a Network Collaborative Decision Making approached is proposed to cope globally with the congestion of a distributed air transportation system. Network performance

measures are developed for each airline, introducing equivalent delay multipliers for its flights while equity concepts, such as non-inferior solutions, between operators are considered at the level of each airport. Then slot allocation algorithms can be designed to manage with more equity and efficiency the arrival and departure capacity at the different airports of the air transportation network.

2 SLOT ALLOCATION FOR AIRPORT TRAFFIC

A slot can be defined as the authorization given by airport authorities to use all the facilities necessary to provide an air traffic service, on a concrete date and time, before a departure or after an arrival. In this section, for sake of brevity only arrival traffic at airports is considered.

2.1 Slot Allocation Practice

The European allocating system considers use rights by companies that have operated in an airport using repeatedly some slots. Airlines can swap these slots with other companies under the rule ''one by one'' without any money transference. If any company under-uses a certain slot, airport may take that slot away from this company by the next season. Some slot re-allocation algorithms have been developed recently to make a better use of slots released by cancellations or by delays. These algorithms convert a nominal schedule into a new one, taking into account the rights of every company and the current traffic situation. In a first step, the air traffic manager will reallocate the flights trying to maximize the use of the available slots and therefore capacity. Then, airlines will try to optimize their position in this by seeking the best arrival slots for its flights. No scheduled flights will be cancelled or delayed.

Let $S = \{s_1, s_2, \dots, s_{n_s}\}$ be a finite set of arrival slots corresponding to the time intervals (s_i= $[t_i, t_i + T]$ where T is a common slot duration), ordered chronologically, i.e., if $i < j$ and $t_i + T \leq t_j \forall\ s_i, s_j \in S$, then slot *i* is before slot *j*: $s_i < s_j$.

Let *A* represents the overall set of airlines operating in the considered airport. A set of no-cancelled flights $F_A = \{f_1^a, f_2^a, \dots, f_{n_s}^a\}$ is associated to each airline *a*. Let $F = \bigcup_{a \in A} F_a$ be the set of arriving flights. Assumptions are the following:

- $n_f = \sum_{a \in A} n_a \leq n_s$, where n_a is the number of arriving flights operated by airline *a*, n_f the overall number of arriving flights and n_s the number of slots at the arrival airport during a considered time period.
- for each aircraft it is possible for the airline to compute the earliest arrival time (e_{AT}) for the flight $f \in F$, expressed in slots. It is written as $h_f \in S$.
- it is not allowed to assign a flight f to a slot s_j if $s_j < e_{AT}$.

The set of earliest arrival times for the considered time period is given by H= $\{h_f, f \in F\}$. A feasible arrival schedule will be an application Π from *F* to *S* that fulfils the following properties:

$$\text{Injective: if } f \neq f' \rightarrow \Pi_{(f)} \neq \Pi_{(f')} \tag{1}$$

$$\text{Feasible:} \quad \forall f \in F, \Pi_{(f)} \geq h_f \tag{2}$$

Airlines have definite user rights over the slots even if they are not their true owners and, at least firstly, there must be a feasible schedule that assign the flights belonging to an airline to the slots this airline has the right to operate in. Let us define the function Φ from *S* to *A* for which it exists Π that fulfils:

$$\forall f \in F, f \in F_a \rightarrow \Phi(\Pi_{(f)}) = a \tag{3}$$

A fair available slots allocation is defined by the pair (Π, Φ) that satisfies the conditions (1), (2) and (3). Taking all of this in consideration, it is necessary to find a reassignment mechanism, called σ, that, given a slot pre-allocation (Π, Φ), produces a feasible assignment H' for which the duo (Π', Φ') is an improved fair allocation from the points of view of the airlines and of the air traffic manager. In the two cases, a reallocation algorithm will be started-up.

2.2 Fair slot allocations

Here it is considered the air traffic manager's objective is be to maximize the available capacity by distributing the available slots among the flights as soon as it is possible to do it. This leads the initial schedule to be compressed by moving some flights forward in the schedule so they can occupy empty slots before theirs. These flights, at the same time, will free the slots they had been previously assigned so other flights might occupy them. A first compression algorithm has been developed by the air traffic management authorities. This algorithm works on an initial slot allocation and the earliest arrival time of flights. It allows not only to assign flights to slots, but also to redefine fairly the slot user-rights of the airlines.

A schedule Π will be preferred over another Π' by airline a, if and only if the programming of its flights is, in its opinion, at least as more favourable ($\Pi'(f) \leq \Pi(f) \quad \forall f \in F_a$), and the position of at least one of its flights is judged by this company as strictly more favourable then it is written: $\Pi' <_a \Pi$.

If a programmed flight is cancelled for any reason, a slot becomes free and the airline in possession of it can decide by itself if any other of its flights is moved forward into this slot, on the condition that this flight can use this slot. The airline can also decide to swap this slot with any other company that could have another slot and may be interested in acquiring the free slot of the first airline. Of course, the air traffic manager is informed of every modification that may occur. In this case, the rights over the exchanged slot will remain in ownership of the first company in the subsequent days as if no exchange had ever taken place.

The first Step towards the satisfaction of the airlines throughout the slot allocation process underlies the so called *non-minimum allocation*. For such allocation, there is no other slot assignment that would be preferred together by all the involved airlines. However there is still the possibility for some companies to agree among themselves to exchange slots at the expense of the others given the limited capacity on landing airport. It is then necessary to define the notion of *non-dominated allocation*.

It is considered that an allocation is non-dominated when it is not possible for a subset of companies (a coalition) to exchange slots among them so none of them loses out and at least one of them gets a strictly better off. More formally, an initial allocation of slots (Π^0, Φ^0) is said non-dominated if there is no other programming Π'and a subset of companies B $(B \subset A)$ such as:

$$\forall f \in \bigcup_{a \in B} F_a : \Phi^0(\Pi'(f)) \in B \qquad \text{and} \qquad \forall a \in B : \Pi' <_a \Pi \tag{4}$$

This negotiation process between airlines could be avoided if the traffic manager directly produces a fair assignment that is non-dominated from the point of view of the airlines. An example of an algorithm that leads to non-dominated assignment is the cyclical algorithm.

The compression algorithm as well as the cyclical algorithm make use of the earliest arrival time information of each flight. It can be shown that airlines have no interest to lie about this data to try to get a better slot allocation, so these algorithms can be considered as collaborative slot assignment schemes. However, an important problem carried with these algorithms is that no consequences regarding what happens in the whole network are taken into account.

3 MODELLING FLIGHT SCHEDULES IN AIRLINES NETWORKS

In this section we introduce a mathematical representation of the flight schedule of an airline to be able to compute cumulative effects in network delays. Here the daily operation of an airline is seen as a set of tasks, the flights, performed successively or in parallel using resources such as aircraft and crews. Aircraft and crews are separately attached to flight pairings covering the whole set of daily flights.

Then let F_a be the set of flights $f \in F_a$ offered by a given airline a, $a \in A$, where A is the set of considered airlines . A schedule published for customers will be such as the one in table 1 informing about origin and destination airports, programmed departure and arrival times. Locally in each airport additional information will be published for passengers: boarding time, boarding terminal and door.

Table 1. Published flights schedule

Flight id	Origin	Destination	Departure local time	Arrival local time
AX123	AZK	BYC	21:34	06:22
BY456	CVT	DWS	22:15	02:17
CZ789	EQR	FGY	23:08	05:09
….	….	….	….	….

The succession of flights is constrained in different ways:

- Flight connections will introduce sequential constraints between flights. These constraints will be effective if there are passengers using these connections.
- Shared aircraft and crew resource will also introduce constraints between flights.

What will not be published is the information about the identity of aircraft and technical and commercial crews which correspond to the airline mobilized resource to perform the flights. In fact feasible pairings of aircraft and crews should be defined to give support to the realization of the flights. Let these airline resources be such as V_a is the set of operated aircraft, C_{Ta} is the set of technical crew and C_{Ca} is the set of commercial crew.

For instance, when considering the aircraft resource, we should have for each flight $f \in F_a$:

$$\sum_{v \in V_a} x_{fv} = 1 \quad \text{with } x_{fv} \in \{0,1\} \tag{5}$$

where x_{fv} is the assignment variable of aircraft v to flight f. This allows to define the set of flights operated by aircraft v:

$$F_{av} = \{f \in F_a, x_{fv} = 1\} \tag{6}$$

where the F_{av}'s built up a partition of F_a.

This implies sequential constraints between the flights of F_{av} according to the order in which these flights are planned to be operated by the airline. Let this sequence be given by $(f_1^v, \cdots, f_k^v, \cdots, f_{|F_v|}^v)$, then between flights k and k+1 of this sequence, time constraints such as:

$$t_d^{pf_k^v} + D_{f_k^v} + GA_{f_k^v} + GD_{f_{k+1}^v} \leq t_d^{pf_{k+1}^v} \quad (7)$$

should be considered, where t_d^{pf} is the planned or published departure time for flight *f*, DF_f is the nominal duration of flight *f*, GA_f is the duration of arrival ground handling for flight *f*, GD_f is the duration of departure ground handling for flight *f*.

Finally , to each flight *f* can be attached a set of predecessor flights Φ_f such as:

$$\forall \varphi \in \Phi_f : t_d^{p\phi} + DF_\phi + GA_\phi + GD_f \leq t_d^{pf} \quad (8)$$

The set of source flights S_a is composed of the flights for which $\Phi_f = \emptyset$ (no predecessor) and the set of end flights E_a is such as (no successors):

$$E \cap \left(\cup_{f \in F} \Phi_f\right) = \emptyset \quad (9)$$

The pair $[F_a, \Phi]$ builts up a directed graph G_a without circuits, then it is possible to assign a rank r_f to each flight *f* where:

$$\forall f \in S_a : r_f = 1, \forall f \in F_a - S_a : r_f = \max\left\{r_\phi, \phi \in \Phi_f\right\} + 1 \quad (10)$$

4 ASSESSING AIRLINE NETWORKS TIMING

4.1 Delay propagation in airlines' networks

For each flight *f* in S_a is attached an earliest starting time t_d^{ef} which correspond to the earliest time at which all resources for this flight can be made available at its departure airport. Then the following condition is satisfied:

$$\forall f \in S_a : t_d^{ef} \leq t_d^{pf} \quad (11)$$

For each flight *f* in E_a is attached a latest starting time t_d^{lf} which correspond to the latest time at which all resources for this flight can be made available at its departure and arrival airports Then the following condition is satisfied:

$$\forall f \in E_a : t_d^{pf} \leq t_d^{lf} \quad (12)$$

Beginning from the earliest starting times of flights in S_a it is possible to compute by increasing rank for each remaining flight an earliest starting time t_d^{ef} given by:

$$\forall f \in F_a - S_a : t_d^{ef} = \max_{\phi \in \Phi_f} \left\{t_d^{e\phi} + DF_\phi + GA_\phi + GD_f\right\} \quad (13)$$

Now, beginning from the latest starting times of flights in E_a, it is possible to compute through decreasing rank for each remaining flight a latest starting time t_d^{lf} given by:

$$\forall f \in F_a - E_a : t_d^{lf} = \min_{\phi \in \Phi_f^{-1}} \left\{t_d^{l\phi} + DF_f + GA_f + GD_\phi\right\} \quad (14)$$

Then for each flight $f \in F_a$, it is possible to compute the feasible period for departure Δ_f given by:

$$\Delta_f = [t_d^{ef}, t_d^{lf}] \tag{15}$$

and a necessary condition for a flight program to be feasible is that:

$$\forall f \in F: t_d^{pf} \in \Delta_f \tag{16}$$

4.2 Cumulative delays in airlines networks

Since no flight will depart before his published departing time, effective margins can be computed from the pairs $\{t_d^{pf}, t_d^{lf}\}, f \in F_a$. They will provide an indication of the robustness of the proposed published flight program. Total margins are given by:

$$\forall f \in F_a : m_{total}^f = t_d^{lf} - t_d^{pf} \tag{17}$$

Free planned margins are given by

$$\forall f \in F_a : m_{free}^f = \min_{\phi \in \Phi_f} \{t_d^{p\phi} - GD_\phi\} - GA_f - t_d^{pf} \tag{18}$$

Let $S_{ad}(t_d^{pf})$ be the set of flights of airline a which have already departed at that time t_d^{pf}, then the expected departure times of flights ϕ of $F_a - S_{ad}(t_d^{pf})$ will be t_d^{ϕ} given through increasing rank as:

$$t_d^{\phi} = \max\{t_d^{p\phi}, max_{g \in \Phi_\phi}\{t_d^g + DF_g + GA_g + GD_f\}\} \tag{19}$$

For flights of $S_{ad}(t_d^{pf})$, t_d^{ϕ} is equal to the effective departing time.

To know the current total effect of a delay δ to departure time of flight f, the above computation should be restarted with t_d^f incremented by δ:

$$t_d^{\phi}(\delta) = \max\{t_d^{p\phi}, max_{g \in \Phi_\phi}\{t_d^g(\delta) + DF_g + GA_g + GD_f\}\} \tag{20}$$

Flights of airline a which can be affected by an additional delay on flight f are those of the closure of Φ^1 applied to f in incidence graph G_a, written $C_{G_a}^f$. Then the total induced delay resulting from an additional delay δ for flight f of airline a will be given by:

$$T_{af}(\delta) = \sum_{f \in S_{ad}} \left(t_d^{\phi}(\delta) - t_d^{\phi}(0)\right) \tag{21}$$

Here according to the way new expected departure times are computed, $T_{af}(\delta)$ should be an increasing piecewise linear function of δ. The maximum sustainable value of δ, written $\delta_{a\max}^f$, is given by:

$$\delta_{a\max}^f = \min\{\delta / \exists \phi \in C_{G_a}^f : t_d^{\phi}(\delta) = t_d^{l\phi}\} \tag{22}$$

Then if departure delay for flight f is greater than $\delta_{a\max}^f$ the remaining flight program is no more feasible and must be revised.

4.3 Effective delay multiplier

In 1998 Beatty introduced the concept of delay multiplier to evaluate the delay propagation in an airport network. Here, different delay multipliers can be defined for the delayed departure of a flight given the current operational situation of the airline *a*'s network:

- a sliding mean delay multiplier written $DM_{af}(\delta)$ and given by:

$$DM_{af}(\delta) = T_{af}(\delta)/\delta \tag{23}$$

- a mean delay multiplier written DM_f^{mean} and given by:

$$DM_{af}^{mean}(\delta) = T_{af}(\delta_{a\max}^f)/\delta_{a\max}^f \tag{24}$$

- a maximum delay multiplier written $DM_{af}^{\max}$ and given by:

$$DM_{af}^{\max} = \max\left\{DM_{af}(\delta), \delta \in]0, \delta_{a\max}^f]\right\} \tag{25}$$

It appears that the importance of an additional delay to a flight *f* can be assessed in different ways using different indexes. One of them is the total induced delay $T_f(\delta)$ given by (21), another one is the closeness to the maximum allowable delay $\delta_{\max}^f$ given by (22) beyond which the airline schedule is no more feasible. Then to get a unique delay index at a given time and place for a flight *f*, the effective delay multiplier (*EDM*) is introduced as an increasing function of the total induced delay and a decreasing function of the current maximum allowable delay:

$$EDM_f = T_{af}(\delta)/\delta_{a\max}^f \qquad f \in F_a \tag{26}$$

5 NETWORK CDM SLOT ALLOCATION ALGORITHM

The consideration of the effective delay multiplier as a mean of ranking concurrent arrival or departure flights within a slot allocation algorithm allows to take into account network effects of slot allocation either at arrival or departure. The effective delay multipliers of the flights are computed on line by the airlines over the subnetwork of flights they operate between the considered airports. Then arrival and departure slot allocation processes can be performed in a decentralized way at the airport level.

5.1Processing at Airports

In this section is described the processing of the string of arriving and departing flights from an airport imbedded in an air transportation network.

It is supposed at at initial time t_0, a nominal solution Π^0 is available with Φ^0. Let the set of airports be given by $K = \{1,2,\ldots,k,\ldots,m\}$. In each airport, departures and arrivals processes are coDnsidered (*p*=1 for departures and *p*=2 for arrivals).

The current time windows for departures and arrivals are considered, they are starting respectively at T_{ID} and T_{IA} and ending respectively at T_{FD} and T_{FA}.

At current time t_c define two time windows (W_D for departures and W_A for arrivals). These time windows start respectively at *iadt* and *iaat*.

For arrivals *iadt*-t_c will correspond to a standard descent duration while the width of W_D will correspond to twenty minutes of cruise flight.

For departures, *iaat*-t_c will correspond to a ground handling and taxiing delay while the width of W_A will be equal to the one of W_D. Let $S_p = \{s_{p1}, s_{p2}, \ldots, s_{pi}\}$ be a finite arrival slots set corresponding

to the schedule p at an airport k and consider the time intervals ($s_{pi} \in [t_i, t_i + T]$), where T is the common slot duration), ordered chronologically.
To each airline $a \in A$ operating at airport k is associated a set of non-cancelled departing and arriving flights: $F_{pa} = \{f_{p1}, f_{p2}, \dots, f_{pn_a}\}$. Let $F_p = \cup_{a \in A} F_{pa}$ be the current considered set of departing and arrivals flights.
The earliest take off time (e_{TOT}) for the flight $f_1 \in F_1$, is written as $h_{f1} \in S_1$. A departing flight f cannot be assigned to a departing slot s_j if $s_j < e_{TOT}$. Let $H_1 = \{h_{f1}, f_1 \in F_1\}$.
The earliest arrival time (e_{AT}) for the flight $f_2 \in F_2$, is written as $h_{f2} \in S_2$. An arrival flight f cannot be assigned to an arrival slot s_j if $s_j < e_{AT}$. Let $H_2 = \{h_{f2}, f_2 \in F_2\}$ and define $\bar{\boldsymbol{H}} = \{H_1, H_2\}$.

5.2 Network slot allocation algorithm

Here the considered slot allocation algorithm is basically a compression algorithm where the effective delay multipliers of each arriving or departing flight allow to rank these concurrent flights in a way to manage airport traffic at the network level. The proposed compression algorithm develops then a greedy heuristic based on the effective delay multiplier attached to each flight. This algorithm is described in broad terms as follows:

Step 1: Start from a nominal initial fair slot allocation: Π^0, Φ^0.

Step 2: Update the time windows W_D and W_A

Step 3: Go to the next schedule on the list. Let $L_p = S_p / \Pi_p(F_p)$ represent the free slots set in the current schedule. If there are no more schedules (departing or arrival), make p=0**.** Go back to Step 2.

Step 4: If $L_p = \emptyset$, end of the calculations: The slot allocation solution will be (Π_p, Φ_p) and go back to Step3. If $L_p \neq \emptyset$, select the earliest free slot $(s_p \in S_p)$ and declare it as active: $c = s_p$.

Step 5: Considering all the flights f_p that fulfil:

1) They occupy a slot, in Π_1, located after c, that is: $(\Pi_p(f_p) > c)$
2) They can use the slot c $(c \geq h_{fp})$

Choose the flight with the higher *EDM* in Π_p. Let s' be its current slot, i.e. $s' = \Pi_p(f_p)$. Go to Step 6.
If it is not the case, erase *c* out of the free slots list $(L_p = L_p / \{c\})$ and go back to Step 4.

Step 6: Move the flight f_p from the slot s' to the slot *c* $(\Pi_p(f_p) = c)$ and transfer the use-rights over the slot *c* to the airline that operates the flight f_p. Assign to *a* the use-rights over the slot *s'*, $\Phi(s') = a$, erase *c* out of L_p, $(L_p = L_p / \{c\})$, and add *s'* to L_p, $(L_p = L_p \cup \{s'\})$. The slot *s'* is declared as active $c = s'$ and we go back to Step 5.
End of the algorithm

6 CONCLUSIONS

In this paper a Network Collaborative Decision Making approached has been considered to cope globally with the operation of a distributed air transportation system. Airlines network performance measures have been developed for arrival and departure traffic management at airports. Equity concepts, such as non-inferior solutions, between operators have been integrated in the decision

scheme when considering airport slot allocation to flights. The proposed slot allocation process operates in a decentralized way by using effective delay multipliers computed by the airlines at the network level to assign priority levels at arriving or departing flights. This should allow to manage with more efficiency and equity the arrival and departure capacity at the different airports composing the air transport network.

REFERENCES

Vossen, T. W. M. (2002) Fair Allocation Concepts in Air Traffic Management. PhD Dissertation, University of Maryland at College Park, USA.

Da Silva Souza M.A., L. Weigang and A. Pereira do Lago, Stable Two-Sided Matching of Slot Allocation in Airport Collaborative Decision Making in Lecture Notes in Business Information Processing 224 · August 2015.

Capgemini Consulting (2010) Our View on Collaborative Decision Making in Aviation.Okwir A. and S.Correas (2014)

Collaborative Decision Making (CDM) in Airport Surface: Europe vs USA Implementations, Challenges and Best Practices, Published in: *Integrated communications, Navigation and Surveillance Conference* (ICNS), ISBN: 978-1-4799-4892-5ACI.

Cook A. and G. Tanner (2011) Modelling the airline costs of delay propagation, *AGIFORS Airlines Operations Conference*, London, United Kingdom, May 16 – 19.

Eurocontrol, IATA, 2012. The Manual. Airport CDM Implementation.Beatty, R., R. Hsu, L. Berry, and J. .Rome (1998) Preliminary Evaluation of Flight Delay Propagation Through an Airline Schedule. *Air Traffic Control Quarterly*, Vol. 7(4), pp. 259-270.

Harary F. (1969), Graph Theory, Addisson-Wesley Series in Mathematics, Addison-Wesley Pub. Co.,Reading, Massachusetts.

F. Mora-Camino (2004), Modélisation, Optimisation , Complexité et Algorithmes, Lectures Notes, CNAM Toulouse.

Beatty, R., Hsu, R., Berry, L., Rome, J., 1998. Preliminary Evaluation of Flight Delay Propagation through an Airline Schedule. Second USA/Europe Air Traffic Management R&D Seminar, Orlando, USA.

Kondo A. (2010) Delay Propagation and Multiplier, *51st Annual Transportation Research Forum*, Arlington, Virginia, March 11-13, USA.

Flores Diaz C. (2015) Airport Capacity Management under Network CDM, Internship report, MAIAA, ENAC, July 2015, Toulouse, France.