Incorporating No-Fly-Zones in Traveling Salesman Problem for Drones: A Case Study in Florianópolis/SC

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ABSTRACT

This study aims to address the Traveling Salesman Problem tailored for drone applications by considering the inclusion of No-Fly-Zones (NFZ). Unlike previous studies, this research incorporates the restricted areas in which drones are not allowed to fly into mathematical formulation. The study's main objectives are to optimize the drone's route in the presence of NFZ, using the city of Florianópolis/SC as the geographic context, and Nauru 500 from XMobots® as the selected drone model, a remote pilot electric vertical takeoff and land (eVTOL) aircraft. The methodology employed comprised several steps. Initially, five points of interest were meticulously chosen. Subsequently, the distance matrix was calculated using the Haversine formula. The optimal route was determined through Mixed Integer Programming (MIP) within the AMPL Tool and Gurobi Solver, incorporating the No-Fly Zone (NFZ) constraint. The suitability of the chosen model to execute the route was assessed against its characteristics. Finally, the obtained route was visualized using the QGIS tool. Our findings showcase an optimized route: Morro da Cruz-Estádio Ressacada-Ilha das Vinhas-Ponte Hercílio Luiz-Ponte Hercílio Luiz-Morro da Cruz. By incorporating DTCEA-FL as a No-Fly Zone (NFZ), the total distance amounts to 20.5 km. Remarkably, this distance aligns with the Nauru 500's endurance, achievable through vertical take-off and vertical landing, or alternatively, parachute-assisted landing. This research advances the comprehension of effective drone route planning with academic, military, and social impacts: addressing overlooked aspects in academia, enhancing military safety and compliance, and promoting safe drone use for privacy, security, and societal integration. Future research holds promise in exploring diverse avenues, including the incorporation of NFZ edges rather than points, investigating a variety of drone models and scenarios, and delving into situations where drones enter temporary NFZ.

Keywords: Air transport, Mathematical model, Optimization.

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1. INTRODUCTION

Drones have witnessed significant growth in various military and civilian applications. However, the presence of NFZ poses unique challenges in planning the route for drones. NFZ are areas where drone flights are restricted due to safety, security, or legal reasons. Existing studies on the Traveling Salesman Problem (TSP) for drones have not explicitly considered the inclusion of NFZ in their mathematical formulations, limiting their practical applicability. This study aims to bridge this gap by introducing NFZ into the TSP formulation for drones. To do this, it will be considered the following objectives:

- Develop a mathematical formulation of the TSP considering NFZ for drone applications.
- Optimize the drone's route in the presence of NFZ in Florianópolis/SC, considering the specific characteristics of the remoted pilot eVTOL Nauru 500 from XMobots®.
- Provide insights and recommendations for efficient route planning for drones in scenarios involving NFZ.

By considering NFZ in the TSP formulation for drones, this research addresses a crucial aspect that has been overlooked in previous research. his incorporation is indispensable to ensure strict compliance with airspace regulations, elevate safety measures, and effectively deter unauthorized drone activities.

With the incorporation of NFZs into the TSP formulation, this study enriches drone route planning. It seeks to offer practical insights for optimizing routes while upholding airspace regulations and avoiding restricted zones. The research outcomes hold potential implications for diverse drone applications, amplifying their safety, efficiency, and legal compliance.

The paper's structure is as follows: Section 2 introduces a comprehensive literature review; Section 3 outlines the employed methodology; Section 4 exhibits the achieved results; and Section 5 concludes, pointing towards future research directions.

2. LITERATURE REVIEW

The Traveling Salesman Problem (TSP) is a combinatorial optimization problem extensively studied in the field of operational research. It involves finding the optimal route solution for a given set of nodes to be visited exactly once, returning from the starting node that minimizes travel cost (Applegate et al., 2011).

In Khoufi et al. (2019) study, various military, and civilian applications of the Traveling Salesman Problem (TSP) as trajectory optimization have been explored, including delivery-transportation, delay tolerance networks, intelligence, surveillance, and reconnaissance (ISR), monitoring, tracking, and filming using drones. However, numerous other potential applications have

emerged with the advancements in drone technology.

The advancements in TSP formulations are studied by Orman and Williams (2006) mathematical formulation of TSP, where there are eight distinct formulations of TSP as Integer Programming compared and their computational result on small problems are studied. Among these, Dantzig et al. (1954) is one of the classical mathematical formulations for TSP. Their formulation incorporates a degree constraint, subtour elimination, and integrality to ensure connectivity. However. the computational complexity of these constraints grows exponentially as the number of nodes increases, thus not solving the problem in a practical way.

In another study, Miller et al. (1960) addressed the issue of dimensionality by proposing a math formulation that introduces additional node potential variables, resulting in a polynomial number of constraints.

To analyze the recent literature that adapted TSP to drone applications in the context of the selection of path and trajectory, Khoufi et al. (2019) analyzed formulations and methods proposed, resulting in a classification and taxonomy of their problem and variations, including using drones with another way of transport as truck, as the study of Murray and Chu (2015).

The NFZ imposes another challenge for TSP by drones but is not considered in the studies above. So, this study considers restricted NFZ, proposing a math formulation of TSP considering NFZ and using the Florianópolis/SC scenario to show results and a Nauru 500 from XMobots as a Drone, as in Table 1.

Table 1 - Literature Review. Source: Research Data.

Author	Main Contribution
Dantzig et al. (1954)	TSP Math Formulation in which computational complexity of constraints grows exponentially as the number of nodes increases.
Miller et al. (1960)	TSP Math Formulation that introduces additional node potential variables, resulting in a polynomial number of constraints.
Orman and Williams (2006)	Advancement in the mathematical formulation of TSP in eight distinct formulations as Integer Programming
Khoufi et al. (2019)	Analyze the recent literature that adapted TSP to drone applications in the context of the selection of path and trajectory
This study	Math formulation of TSP considering NFZ, using Florianópolis/SC Brazil and XMobots Nauru 500 as scenario.

3. METHODOLOGY

In this section, it is presented the methodology employed to address the problem of optimizing drone routing considering NFZ.

As part of the initial phase, five points were carefully selected in Florianópolis/SC. These points include: Morro da Cruz (-27.5879, -48.5337), Ponte Hercílio Luiz (-27.5937, -48.5659), Destacamento de Controle do Espaço Aéreo de Florianópolis/SC - DTCEA-FL (-27.6665, -48.5411), Estádio Ressacada (-27.6663, -48.5316), and Ilha das Vinhas (-27.6145, -48.5513).

The selection was based on their significance for surveillance and delivery purposes in military and civilian applications. Additionally, it is worth noting that DTCEA-FL is designated as an NFZ, further emphasizing its importance in terms of airspace control and regulations.

Next, from the decimal latitude and longitude of two points, and radius of the earth R = 6371.009 km (Azdy and Darnis, 2020), the distance Great Circle Distance (GCD) is evaluated using Haversine Formula

$\text{GCD (km)} = \text{R (km)} * \theta \text{ (radians)}$ (1)

$\theta =$

$2 \sin^{-1} \sqrt{\sin^2 \Delta L + cosL_1 * cosL_2 * \sin^2 \Delta G}$ (2) ,

where L_1 is the latitude of first point (radians); L_2 is the longitude of second point (radians); ΔL is the half difference of two points in Latitude (radians); ΔG is the half difference of two points in Longitude (radians).

From the considered locations and haversine formula, the distance matrix is generated as in Table 2.

Table 2 - Distance Matrix. Source: Research Data.

	Morro da Cruz	*Ponte Hercílio Luiz*	*DTCEA-FL*	*Estádio Ressacada*	*Ilha das Vinhas*
Morro da Cruz	0,00	3,10	8,77	8,72	3,39
Ponte Hercílio Luiz	3,10	0,00	8,43	8,72	2,65
DTCEA-FL	8,77	8,43	0,00	0,94	5,90
Estádio Ressacada	8,72	8,72	0,94	0,00	6,10
Ilha das Vinhas	3,39	2,65	5,90	6,10	0,00

Using the identified Locations (L), NFZ, calculated Drone Distance Matrix ($d_{i,j}$), a drone routing solution is generated, considering the restrictions imposed by the NFZ, as in Figure 1.

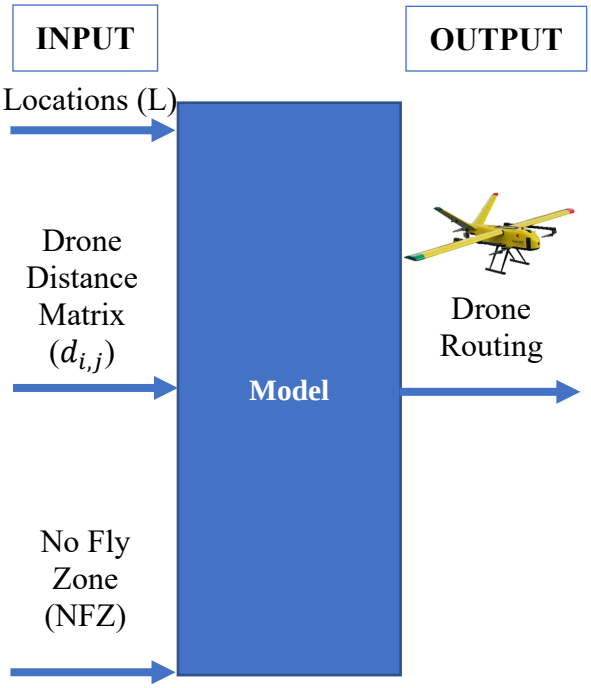


Figure 1 - Black Box Diagram of the Model. Source: Research Data.

The model described in Figure 1 is implemented using the mathematical programming language AMPL and Solver Gurobi, where from the model file and data file, a solution for drone routing with minimum distance considering NFZ is calculated, as can be shown in Figure 2.

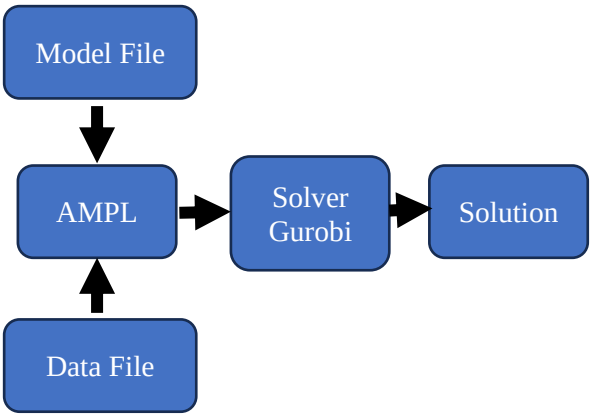


Figure 2 -Methodology. Source: Research Data.

To validate the feasibility of the drone routing solution using real equipment used in both civilian and military applications, the Nauru 500 drone is employed. The choice of the Nauru 500 is because the Brazilian Army possesses a drone from the XMobots Nauru Family, and it is also part of a collaborative project between XMobots and CEI-ITA. By utilizing the Nauru 500, the practical implementation and performance of the proposed solution can be assessed, ensuring its applicability in real-world scenarios.

Table 3 - Drone characteristics. Source: XMobots (2023).



Drone Nauru 500 Characteristics	
Autonomy	40 h
Endurance	60 km
Payload	25 Kg
Power Capacity	Battery and Combustion
Take off and Land	Vertical Take-Off and Land (VTOL) with the inclusion of a parachute as an alternative landing method

From these characteristics in Table 3, autonomy, endurance, take-off, and land will be verified in the routing solution.

4. RESULTS

The following section presents the results of the study, including a math model symbol description, math model formulation of the TSP variant considering NFZ, an optimal route solution in the QGIS tool, and drone route validation.

The mathematical model, illustrated in Figure 1 and Figure 2, utilizes the following parameters: $d_{i,j}$ represents the distance between source location i and destination location j, which is evaluated using the Haversine formula (1) and (2). N denotes the total number of locations involved. Within the mathematical mode, the binary variable $x_{i,j}$ is employed to determine connections between locations i and j, in accordance with the objectives and constraints. Additionally, the auxiliary variable u_i is utilized to make decisions based on the Miller-Tucker-Zemlin (MTZ) approach for subtour elimination. The sets L, NFZ, and FL are employed to describe all locations, locations within NFZ, and allowed locations, respectively. These details are summarized in Table 4.

Table 4 - Math Model Symbol Description.

Source: Research Data

PARAMETERS	
$d_{i,j}$	Geographical distance between locations i and j
N	The cardinality of the set Locations
DECISION VARIABLE	

$x_{i,j}$	Binary variable denoting the presence of a link between locations *i* and *j*
AUXILIARY VARIABLE DECISION	
u_i	Sub-tour elimination integer variable based on the Miller-Tucker-Zemlin approach
SETS	
L	Set of Places
$sNFZ$	Set of No-Fly-Zone Places
FL	Set of allowed location
INDEX	
i, j	Source and Destiny indexes

Subsequently, the mathematical models strive to minimize the distance covered, as indicated by Equation 3, while being subject to:

- Constraint of "arriving and departing once" guarantees that every location is visited and departed from exactly once, as illustrated in Equation 4.
- Constraint of "departing from and returning to a fixed point" rigorously mandates the flight to initiate and conclude at a designated location, as explicitly stated in Equation 5.
- Constraint of the NFZ sternly forbids any connections to or from the specified restricted airspace, encapsulated within Equation 6.
- Constraint to incorporate subtour elimination, based on Miller-Tucker-Zemlin (MTZ) formulation (Equation 7), effectively eradicating sub-tours formation.
- Finally, the variables are defined as: decision variable $x_{i,j}$, representing a binary variable that indicates the existence of a connection between locations i and j. The variable u_i represents an integer in the model. And the initial integer variable, u_1, is constrained to have a fixed value of 1, as in Equation 8.

$$\min \sum_{i,j \in L} d_{i,j} * x_{i,j} \quad (3)$$

$$\sum_{j \in L, i \neq j} x_{i,j} = \sum_{i \in L, i \neq j} x_{j,i} = 1 \ \forall i \in FL \ (4)$$

$$\sum_{i \in L} x_{i,1} = \sum_{j \in L} x_{1,j} = 1 \ (5)$$

$$x_{i,j} = x_{j,i} = 0 \ \forall i \in L, \forall j \in sNFZ, i \neq j \ (6)$$

$$u_i - u_j + N * x_{i,j} \leq (N-1), \forall i, j \in L, i \neq j \quad (7)$$

$$x_{i,j} \in \{0,1\}, u_i \in Z^+, u_1 = 1 \ (8)$$

The goal of this mathematical model is to minimize the distance expressed by Equation 3, while subjected to constraints outlined in Equations 4 to 8.

The constraint in Equation 4 ensures that for each location in the feasible locations (FL) excluding the NFZ, there's exactly one edge coming into the location and one edge leaving the location. This means that each location is visited only once.

Equation 5 mandates that exactly one edge is leaving from location 1 and exactly one edge is returning to location 1. This ensures that the salesperson's journey starts and ends at location 1.

Equation 6 enforces that there are no edges between cities in the NFZ and any other location. Additionally, Equation 7 presents a part of subtour elimination methods. They ensure that if there is a connection between locations, the auxiliary variable u_i minus u_j, adjusted by N times $x_{i,j}$, is at most N−1. This helps prevent the formation of smaller disconnected cycles within the larger tour.

Equation 8 defines the nature of the variables: $x_{i,j}$ is a binary variable that indicates the presence (1) or absence (0) of an edge between locations; u_i is a non-negative integer auxiliary variable; $u_1 = 1$ initializes the auxiliary variable for Location 1 – Morro da Cruz.

These equations together form a mathematical representation of the Traveling Salesman Problem (TSP) with an additional constraint of NFZ. The goal is to find the optimal arrangement of edges (connections) that satisfies these constraints and minimizes the total distance traveled.

Following the methodological procedure in Section III, the model successfully attained the optimal solution of 20.5 km. This accomplishment was attained by considering the route Morro da Cruz – Estádio Ressacada (8.7 km), Estádio da Ressacada-Ilha das Vinhas (3.1 km), Ilha das Vinhas- Ponte Hercílio Luiz (6.1 km), and Ponte Hercílio Luiz-Morro da Cruz (2.6 km), as Illustrated in drone path of Figure 3.

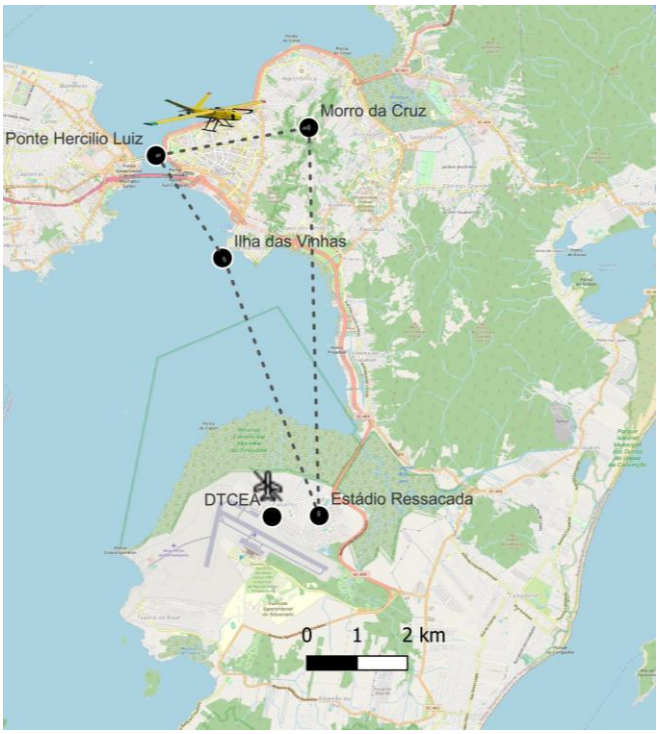


Figure 3 - Drone Routing Solution. Source: Research Data.

The 20.5 km route is well-suited for the Nauru 500, thanks to its 60 km endurance and 40-hour autonomy.

The combination of VTOL and Parachuting Land shows advantages,

particularly due to the conditions at Morro da Cruz for takeoff and landing, a scenario where conventional runways or landing areas are unavailable, difficult, or impractical.

5. CONCLUSION

In this study, it is addressed the challenge of incorporating NFZ in TSP formulation for drones. It achieved the objectives with a proposal of a mathematical formulation that integrates NFZ into the TSP model, enabling efficient route planning for drones while ensuring compliance with airspace regulations; an optimization of the drone's route in the presence of NFZ in Florianópolis-SC with a Drone XMobots Nauru 500, employing optimization techniques to minimize the total distance traveled by the drone while avoiding the restricted area in Florianópolis-SC, offering practical insights and recommendations for optimizing drone routes, enhance safety, and complying with legal requirements in real-world applications.

Future research presents opportunities to delve into multiple avenues, encompassing the integration of NFZ edges into mathematical formulations, the exploration of varied drone models and scenarios, and the examination of cases involving drones entering temporary NFZs.

This research has academic, military, and social impacts. In academics, it contributes to the address of a crucial aspect that has been overlooked in previous research, which is the inclusion of NFZ in TSP formulation. In the military, it contributes to enhancing safety, complying with airspace regulations, and preventing unauthorized drone activities in restricted areas. In social, it lies in promoting the safe and responsible use of drones in civilian applications, ensuring the privacy and security of individuals, preventing disruptions in sensitive areas, and facilitating the integration of drones into aerial surveillance, disaster management, and delivery services.

In conclusion, this study achieved its objectives by developing a mathematical formulation, optimizing drone routes in the presence of NFZ, and providing practical insights for efficient route planning.

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